Heat transfer and interfacial temperature of two-layered convection: Implications for the D″-mantle coupling

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[1]   We performed laboratory measurements of the heat flux and the interfacial temperature of two-layered thermal convection. Experiments show that the scaling law of the heat flux (Nu-Ra relation) can be classified by the buoyancy number B. The heat flux under $B > 1$ follows the well-known Nu-Ra relation. However, under $B < 1$, undulations develop at the interface, and heat flux is systematically enhanced. The interfacial temperature is determined only by the ratio of physical properties between the upper and lower layers. Our results suggest that the existence of less viscous and slightly denser D″ than the overlying mantle would enhance the heat transfer from the core to the mantle.   *INDEX TERMS:* 1213 Geodesy and Gravity: Earth's interior—dynamics (8115, 8120); 5418 Planetology: Solid Surface Planets: Heat flow; 8121 Tectonophysics: Dynamics, convection currents and mantle plumes; 8124 Tectonophysics: Earth's interior—composition and state (8105); 8130 Tectonophysics: Evolution of the Earth: Heat generation and transport. **Citation:** Namiki, A., and K. Kurita, Heat transfer and interfacial temperature of two-layered convection: Implications for the D″-mantle coupling, *Geophys. Res. Lett.*, *30*(1), 1023, doi:10.1029/2002GL015809, 2003.

1.   Introduction

[2]   The D″ region at the base of the Earh's mantle has been regarded to be a compositionally separated layer from the overlying mantle [e.g., *Lay et al.*, 1998; *Garnero*, 2000]. Recent seismological observations and geo-dynamical investigations have suggested that the existence of small-scale convection inside D″ [*Olson et al.*, 1987; *Vidale and Hedlin*, 1998]. Since, such a thermo-chemical basal boundary layer will affect the thermal state of mantle convection, the behavior of this layer has been explored. It has been shown that the layer suppresses the heat transfer from the core to the mantle [*Montague and Kellogg*, 2000], decreases the excess temperature of mantle plumes [*Farnetani*, 1997], and modifies the convection pattern of the mantle [*Tackley*, 1998]. On the other hand, the dynamics of two-layered mantle convection has also been explored by a number of authors [e.g., *Richter and McKenzie*, 1981; *Olson*, 1984; *Davaille*, 1999]. However, the heat transfer and temperature profiles have not been measured quantitatively, because most of the previous works were investigated using two miscible fluids. Through mixing, the structure of convection evolves with time, so the experiments did not reach a steady state.

[3]   To estimate the heat flux and the thermal state of the Earth, the relation between the Nusselt (Nu) and Rayleigh number (Ra), Nu $\sim \gamma \mathrm{Ra}^\beta$ measured under one-layer convection, has been used, where γ and β are experimentally determined constants. Here, the Rayleigh number is defined by

$$\mathrm{Ra} = \frac{\alpha g \Delta T L^3}{\kappa \nu}, \qquad (1)$$

where α is the thermal expansion coefficient, g is the gravitational acceleration, ΔT is the temperature difference in a convecting layer, L is the thickness of a convective layer, κ is the thermal diffusivity, and ν is the kinematic viscosity. For layered convection, thermal and viscous couplings would arise between the two layers, and a topography can develop. These factors would modify the Nu-Ra relation. In particular, we note that topography can have an important effect because it is known that the topographical heterogeneity at the boundary in one-layer convection changes γ and β [*Ciliberto and Laroche*, 1999; *Du and Tong*, 2000]. Here, we infer that the topographical coupling develops between the mantle and D″. The thickness of D″ has been observed seismically [*Kendall and Shearer*, 1994; *Wysession et al.*, 1998], and its lateral variation is sufficient to affect the style of mantle convection [*Namiki and Kurita*, 2001]. In this paper, we quantitatively measure the heat flux and the interfacial temperature of two-layered convection and apply the results to the mantle and D″.

2.   Experimental Settings

[4]   The experimental conditions are given in Table 1. Cases 1–8 are conducted under $B > 1$, and cases 9–12 are conducted under $B < 1$. Here, $B = \Delta\rho/\rho\alpha\Delta T_o$ is the buoyancy number that shows the ratio of compositional stability to thermal buoyancy, where ρ is the density of each layer, $\Delta\rho$ is the density difference between the upper and lower layers, and ΔT_o is the superposed temperature difference between upper and lower boundaries. In cases 1–5, we vary the thickness ratio of two layers, using the same combination of fluids. In cases 2, 6–8, and 11, we changed the working fluid to examine the effect of the variety of physical properties while holding the thickness ratio constant. Except for case 6, the Prandtl number is always greater than 100; i.e. the experiments are conducted under the regime dominated by viscosity [*Krishnamurti*, 1973]. We use two immiscible fluids for layered convections: silicone or castor oil for the

Table 1. Experimental Conditions

Case	lower	upper	Ra_1	Ra_2	α_1/α_2	Cp_1/Cp_2	η_1/η_2	k_1/k_2	ρ_1/ρ_2	L_1'	B_2	B_1	symbols
1	g	s	7.4×10^6	4.5×10^6	0.40	1.7	3.2	2.5	1.3	0.69	11	22	□
2	g	s	6.6×10^6	2.9×10^7	0.40	1.7	3.2	2.5	1.3	0.55	11	22	○
3	g	s	7.3×10^6	2.5×10^8	0.40	1.7	3.2	2.5	1.3	0.39	10	19	◇
4	g	s	4.4×10^5	2.3×10^8	0.40	1.7	3.2	2.5	1.3	0.21	10	19	×
5	g	s	2.4×10^3	2.7×10^8	0.40	1.7	3.2	2.5	1.3	0.044	10	19	+
6	w	s	7.4×10^6	4.5×10^6	0.20	2.5	0.11	4.4	1.1	0.55	4.3	20	*
7	h1	s	7.3×10^6	2.5×10^8	0.20	2.5	1.1	4.4	1.1	0.53	4.3	20	▽
8	h2	s	4.4×10^5	2.3×10^8	0.20	2.5	2.1	4.4	1.1	0.55	5.0	23	◁
9	e	c	2.2×10^7	3.8×10^5	0.71	2.2	0.043	2.8	1.006	0.72	0.44	0.62	
10	e	c	5.6×10^7	5.9×10^5	0.71	2.2	0.022	2.8	1.006	0.72	0.23	0.32	▷
11	e	c	1.4×10^7	1.6×10^6	0.71	2.2	0.022	2.8	1.006	0.53	0.20	0.28	●
12	e	c	2.0×10^7	2.4×10^6	0.71	2.2	0.029	2.8	1.006	0.36	0.18	0.25	△

g: 82 wt% glycerol solution. w: water. h1: 0.49 wt% hydroxyethyl cellulose solution. h2: 0.98 wt% hydroxyethyl cellulose solution. e: 24 wt% ethanol and 0.88wt% hydroxyethyl cellulose solution. s: silicone oil. c: castor oil.
$L_1' = L_1/(L_1 + L_2)$.

upper layer, and water, glycerol-, and ethanol solution for the lower layer. In order to independently vary the viscosity, we add small amounts of hydroxyethyl cellulose. The Rayleigh number at each layer is varied by changing the working fluid, by the height of the convecting layer, and by the temperature difference between the upper and lower boundaries.

[5] The experiment is conducted in a vertical cylindrical cell. The sidewall of the cell is made of acrylic plastic with an inner diameter of 260 mm. The height of the convection cell is changeable. The upper and lower boundaries are made of aluminum plates. A silicone rubber film heater is installed on the backside of the bottom plate. AC power is supplied to maintain the temperature at the bottom plate. From the applied power, we calculate the heat flux. The upper side of the convection cell is in contact with a cooling chamber, whose temperature is maintained by circulating cold water from a temperature-controlled bath. Three small movable thermistor probes are placed inside the cell. The probes are mounted on a stepping motor so that the local temperature of the fluid can be measured as a function of the distance away from the upper and lower boundaries with an accuracy of 0.01 mm. The vertical temperature profiles are determined from the time-averaged temperature measurements at respective heights. As an interfacial temperature, we adopt the temperature at the intersection of two temperature profiles measured from the upper and lower boundaries. Convection patterns are simultaneously recorded using nylon tracers.

3. Results and Discussion

[6] In Figure 1, we show the results of heat-flux measurements for different values of B. For $B > 1$, the two layers are stably stratified, and no topography develops at the interface. However, for $B < 1$, a topography develops as a result of thermal buoyancy (Figure 2). The Nusselt numbers measured under $B > 1$ show good agreement with previous studies based on one-layer convection. The Nusselt numbers under $B < 1$, however, show a different curve, where $\beta = 0.30$ is the same as that under one-layer convection but the prefactor $\gamma = 0.25$ is almost twice that of one-layer convection.

[7] To understand why the heat transfer was enhanced only in the cases where $B < 1$, we studied the flow structure at the interface in detail (Figure 2). Note that the topographic rise at the interface corresponds to the steady upwelling in both the upper and lower layers. This is because a topographical rise introduces a high-temperature

anomaly above the interface, and the resulting lateral thermal anomaly is unstable it becomes a site of upwelling. The steady upwelling simultaneously introduces the lateral temperature anomaly and a positive feedback arises between the topographical- and thermal couplings. Such an upwelling can continuously transfer the heated fluid like a pipe flow. Thus, the steady coupled upwelling would transfer the heat efficiently. On the other hand, for $B > 1$, neither thermal- nor viscous couplings occur because of larger discrepancies in the Rayleigh numbers of two layers. Only when the Rayleigh numbers of the upper and lower layers are similar within a factor of 10 do thermal couplings occur intermittently. However, the coupled structure breaks up within a time scale that is less than the time scale of the convective overturn.

[8] Note that the enhanced heat flux under $B < 1$ cannot be explained by the increase of the surface area resulting from the topography. We varied B in the range of $0.18 < B <$
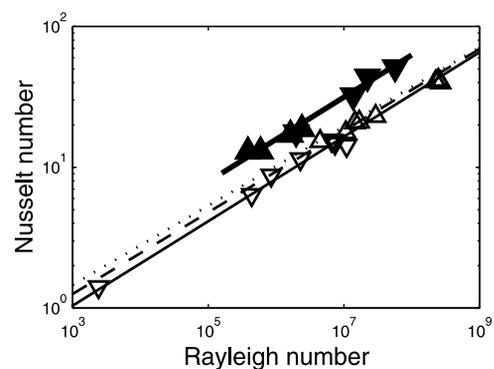
Figure 1. The Nusselt number as a function of the Rayleigh number. The downward and upward triangles show Nu for the lower and upper layers, respectively. The solid triangles indicate $B < 1$, and the open ones indicate $B > 1$. To calculate the Nusselt and Rayleigh numbers, we use the measured interfacial temperature. The thin solid, dashed, and dotted lines describe the Nu-Ra relation of $\gamma = 0.131$, $\beta = 0.30$, $\gamma = 0.17$, $\beta = 0.29$, and $\gamma = 0.21$, $\beta = 0.281$, respectively [*Richter et al.*, 1983; *Du and Tong*, 2000; *Manga and Weeraratne*, 1999]. The thick solid line shows the data fit curve of Nusselt numbers when $B < 1$. Here, $\gamma = 0.25$ and $\beta = 0.30$. The fitting curve for $B > 1$ is written by $\gamma = 0.16$ and $\beta = 0.29$. The average enhancement of the heat transfer in the range of $10^5 < Ra < 10^8$ is 1.85.
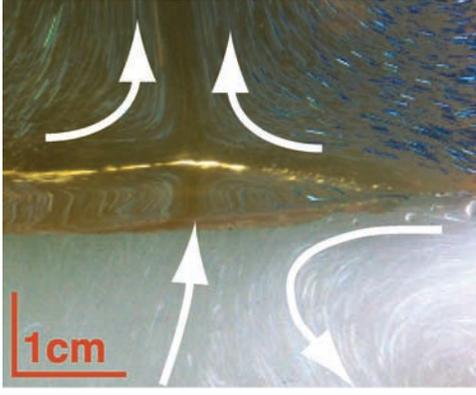
Figure 2. Streak image of the area near the interface for $B < 1$. The area shining in yellow is the ridge of the topography. The surface of the interface reflects the streak image of the upper layer.

0.62. Since the shape and the size of the interface topography depends on B, the contact area should vary as a function of B. Figure 1, however, shows systematic enhancement of the Nusselt number regardless of B, which suggests that the contact area is unimportant for the enhancement of the heat flux. The topography at the interface also increases the effective height of convection for each layer. However, the increase of the convection height cannot explain the enhancement of the heat flux. The maximum increase of the convection height was 20%, which corresponds to the 70% increase of the Rayleigh number and 20% increase of the Nusselt number. The measured increase of the Nusselt number reached 85%, which is much larger than that which can be explained by the convection height. Thus, we suspect that the excess heat flux under $B < 1$ is due to the steady coupling between two layers.

[9] It is known that the roughness at the boundary with variable sizes in height and width modifies the exponent β, but roughness with just one size changes only the prefactor γ under one-layer convection [*Ciliberto and Laroche*, 1999; *Du and Tong*, 2000]. In two-layered convection, the size of the interface topography will be determined by the size of each plume. Here, the size of plumes are determined by the thickness of the boundary layer. Thus, the interfacial topographies have the same size and do not modify the exponent β.

[10] Knowing the Nu-Ra relation for layering convection, we can estimate the interfacial temperatures and compare them to measured ones. The most simple requirement of two-layered convection is that the total heat flows Q in the upper and lower layers should be equal, which is expressed as

$$Q_1 \sim Q_2. \tag{2}$$

Here, suffix 2 means the upper layer, 1 means the lower layer. Using Fourier's law, Q is expressed by $Q = k\Delta TS/\delta$ where k is the thermal conductivity, δ is the thickness of the thermal boundary layer, and S is the area of the convection layer. The thickness of the thermal boundary layer corresponds to $L/2Nu$ [*Belmonte et al.*, 1994]. We already confirmed that the Nusselt number is also written in the form of Nu $\sim \gamma Ra^\beta$ under two-layered convection, so that the thickness of the thermal boundary layer can be
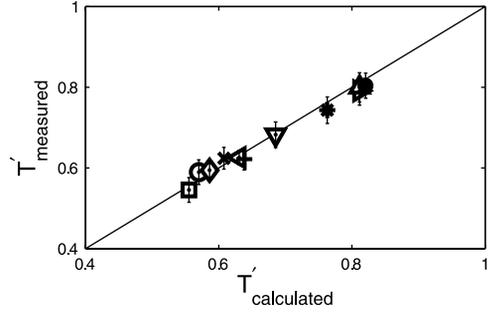
Figure 3. Correlation between the calculated and measured temperature. The interfacial temperature is normalized by the superposed temperature difference between the upper and lower boundaries. The interfacial temperature is calculated by equation (3) where $\beta \sim 0.29$ for $B > 1$ and $\beta = 0.30$ for $B < 1$. The range of the error bar indicates the difference among three thermistor probes. Markers correspond to Table 1.

expressed in the form of $\delta \sim L/2Nu \sim L/2\gamma Ra^\beta$. Arranging equation (2) for $\Delta T_2/\Delta T_1$ using Fourier's law and equation (1) yields,

$$\frac{\Delta T_2}{\Delta T_1} \sim \left(\frac{\alpha_1 C_{p1}\eta_2}{\alpha_2 C_{p2}\eta_1}\right)^{\frac{\beta}{1+\beta}} \left(\frac{k_1}{k_2}\right)^{\frac{1-\beta}{1+\beta}} \left(\frac{\rho_1}{\rho_2}\right)^{\frac{2\beta}{1+\beta}} \left(\frac{L_2}{L_1}\right)^{\frac{1-3\beta}{1+\beta}}, \tag{3}$$

where C_p is the specific heat, and η is viscosity. Here, this equation does not include the prefactor γ, and $\beta \sim 1/3$ is also realized in two-layered convection. Thus, equation (3) shows that the temperature at the interface is almost independent of the variation of the thickness ratio between two layers, but is sensitive to the ratio of physical properties.

[11] Figure 3 shows the correlation between the calculated and measured interfacial temperature. This figure
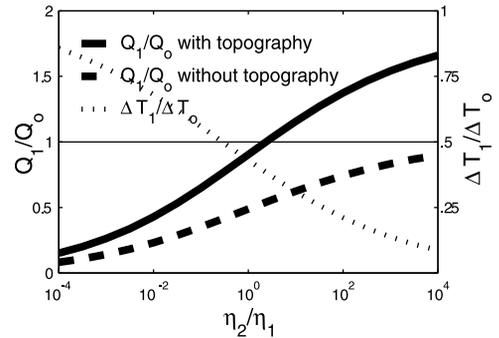
Figure 4. The heat-flux ratio of two-layered (mantle-D″) convection to that of one-layer mantle convection without D″ as a function of the viscosity ratio. The ratio of $\Delta T_1/\Delta T_o$ is estimated using equation (3) and indicated by a dotted line. The ratio of Q_1/Q_o without topography is calculated by equation (4) and is denoted by a dashed line. Our measurements show that topographical coupling enhances the heat flux 1.85 times, and accordingly, the heat-flux ratio for a case with topographical coupling is calculated by multiplying by this factor and is shown by a solid line. Here, we assumed that $\beta = 0.29$, the thickness ratio of the mantle to D″ is 2700/200 (km), and the upper to lower density ratio $\Delta\rho_2/\Delta\rho_1$ is 0.996 [*Kesson et al.*, 1998].

shows good agreement between the calculated and the measured interfacial temperatures. Simultaneously, the interfacial temperatures show their variations as a function of the ratio of physical properties between the upper and lower layers (Table 1). Some experiments in Figure 3 are conducted under the condition of $B < 1$, where the interface shows some undulations. Regardless of B, the scaling law of the equation (3) is realized.

4. Implications for Earth's D″

[12] In general, the layering of convection decreases heat transfer. However, our results indicate that two-layered convection with interfacial topography can transfer more heat than one-layer convection. Here, we propose that such excess heat transferring occurs in the mantle and D″.

[13] Using Fourier's law, the heat-flux ratio of two-layered convection without topography at the interface, to one-layer convection can be expressed as

$$\frac{Q_1}{Q_o} = \frac{k_1 \Delta T_1 / \delta_1}{k_o \Delta T_o / \delta_o}, \qquad (4)$$

where suffix 1 denotes the development of two-layered convection and its lower layer and o denotes one-layer convection. The physical properties of the upper (mantle) and lower (D″) layers are estimated to be the same except for viscosity and density. For one-layer convection, the physical properties are assumed to be the same as those of the upper layer. Thus Q_1/Q_o is written as

$$\frac{Q_1}{Q_o} = \left(\frac{\Delta T_1}{\Delta T_o}\right)^{1+\beta} \left(\frac{\rho_1}{\rho_o}\right)^{2\beta} \left(\frac{L_o}{L_1}\right)^{1-3\beta} \left(\frac{\eta_o}{\eta_1}\right)^{\beta}. \qquad (5)$$

Here, the ratio of the temperature difference $\Delta T_2/\Delta T_1$ is estimated by equation (3), and the imposed temperature difference, $\Delta T_o = \Delta T_1 + \Delta T_2$, is identical. Thus, we can estimate the ratio of heat flux Q_1/Q_o and that of the temperature difference $\Delta T_1/\Delta T_o$ as shown in Figure 4.

[14] Figure 4 shows that the smaller viscosity of the lower layer makes the temperature difference in the lower layer smaller; as a result, the heat transferred by two-layered convection is slightly decreased compared to that of one-layer convection. If topography develops at the interface, the total transferred heat exceeds that of one-layer convection. Applying the scaling in our experiments to the Earth, it shows that, if the viscosity of D″ is 10^{-1} times less than that of the mantle, and the topographical coupling develops between the mantle and D″, the total transferred heat can exceed that transferred by one-layer mantle convection without D″. If D″ is a compositionally dense layer and exists above the core-mantle boundary for a long time, D″ is hot and less viscous than the overlying mantle because of strongly temperature dependent viscosity. Actually, *Panasyuk and Hager* [2000] suggested less viscous D″ by a joint inversion of Earth's geoid and dynamic topography for radial mantle viscosity structure.

[15] It has been known that the estimated current global heat loss exceeds the sum of the heat production by radioactive elements and the secular cooling of the Earth [*Kel-*

logg et al., 1999; *Helffrich and Wood*, 2001]. The source of this discrepancy might be the presence of a dense but less viscous D″ layer at the base of the mantle.

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